Nature & Environment
Vol. 22 (2), July 2017: 44-48
Website: www.natureandenvironment.com

ISSN (Print) : 2321-810X
ISSN (Online) : 2321-8738

RESEARCH ARTICLE

Multiple Vacation and Single Vacation in the Model of The M*/G/1 Queueing System

Anil Kumar Sharma and J.S. Chaudhary
Department of Mathematics, Ganjdundwara College, Ganjdundwara
Email: aksmps@gmail.com

Received: 19th April 2017, Revised: 10th June 2017, Accepted: 15th June 2017

ABSTRACT

We consider an M/G/1 queuing system where the withies at customers are assumed to arrive the system according to a compound Poisson process. As soon as the system becomes empty the server takes a vacation for a random length of time called vacation time to do other jobs, which is uninterruptible. After returning from that vacation, there are two possibilities viz. (i) he keeps on taking vacation till he finds at least one unit in the quene (multiple vacations) or(ii) he may lake only one vacation between two successive busy periods (single vacation). The steady state ehaviour of this M*/G/1 queuening system is derived by an analytic approach to study the queue size distribution at a stationary (random) as well as a departure point of time under multiple vacation policy. Also, attempts have been made to obtain the queue size distribution of a more generalised model at a departure point to cover both the cases multiple and single section.*

Key words: *The M*/G/S. Queue system, vacation time, multiple vacation, single vacation, Queue size*

INTRODUCTION

The queueing system when the server becomes idle is not new. Miller (1964) was the first to study such a model, where the server is unavailable during some random length of time (referred to as vacation) for the M/G/1 queueing system. The M/G/1 queueing models of similar nature hae also been reported by a number of authors, since Levy and Yechiali (1975) included several types of generalizations of the classical M/G/1 queueing system. These generalizations are useful in model building in many real life situations such as digital communication, computer network and production inventory system (Takagi, 1991 and Doshi1986, 1990).

The major general result fir vacation model is the stochastic decomposition result, which allows the system to be analysed by considering separately the distribution of the queue size with no vacation and the additional queue size he to vacation. This important result was first established by Futermann and Cooper 91985D for generalized vacation as well as multiple vacation models, where the servers keeps on taking a sequence of vacations of random length till it finds at least one unit in the system to start each busy period for the M/G/1 queueing system. Later Doshi (1986) extended this result for the single vacation model through a different approach where the server takes exactly one vacation at the end of each busy period.In this model if the server finds no units after returning from a vacation, he stays in the system waiting fora unit to arrive.

Shanti Kumar (1988) showed that the queue size decomposition holds even for the M/G/1 models with bulk arrival, reneging, balking etc. in terms of unfinished work in the system Boxma and Groenendijk (1987) proved the decomposition result for the M/G/1 type vacation models Recently Doshi (1990b) and Leung (1992) extended the results of Boxma and Groendijk (1987). Harris and Marchal (1988) and Shantikumar (1988) proved the stochastic decomposition result for unfinished work in the system and additional delay due to the vacation times respectively in more general setting.

At present, however, most studies are devoted to batch arrival queues with vacation because of its interdisciplinary character. Considerable efforts have devoted to study these models by Baba(1986). Lee and Srinivasan (1989).Leee,*at at.*, (1994, 1995).Borthakur and Chaudhary

(1997) and Choudhary (1998, 2000) among others. However, the recent progress of M*/G/1 type queueing models of this nature have been served by Chae and lee (1995) and Mehdi (1997).

In this chapter, we first study the steady state behaviour of the queue size distribution for this M*/G/1 queue with multiple vacation policy at the stationary point of time as well as departure point of time through an analytical approach. Also we show that the departure point queue size distribution of this model can be expressed as convolution of the distributions of three independent random variables, one of which of the queue size of the standard m*/G/1 queue without vacations. Efforts have also been made to interpret other two random variables properly. Using these interpretations we then derived the probability generating function (pgf) of the departure point queue size distribution of a more generalizd M*/G/1 type vacation models. Moreover, we obtain some important performance measures of these models, which may lead to remarkable simplification when solving the rather complicated vacation models of this nature.

M*/G/1 QUEUEING SYSTEM WITH MULTIPLE VACATIONS:

In this section, we first write the system state equations for its stationary (random) queue size (including the one in service, if any) distribution by treating elapsed service time and vacation time as supplementary variables. We now define the following notations and probabilities:

λ = rate of arrival of batch

X = size of arrival (a random variable)

$$C_k = \Pr[X = k], k \geq 1$$

$X(z)$= *pgt* of X

B = Service time random variable

V = Vacation tie random variable

$B(X)$, $V(X)$ = Probability distribution functions of B and V

$B(x)V(x)$ = Laplace Stieltjes transformations of B(X), V(X)

Further it is assumed that-

$V(0) = 0$ $V(\infty) = 1$ $B(0) = 0$ $B(\infty) = 1$

And that V(x) and B(x) are continuous at X = 0.

So that,

$$V(X)dx = \frac{dV(X)}{[1 - V(X)]}$$

and $$B(X)dX = \frac{dB(X)}{[1 - B(X)]}$$

are the first order differential equations of V and B respectively.

Let $N_0(t)$ be the queue size at the time t and $B^0(t)$ be the elapsed service time at time t. Further, we consider that $V^0(t)$ is the elapsed vacation time at t. Let us now introduce the following random variables.

$$Y(t) = \begin{cases} 0, \text{if the server is idle at time } t \\ 1, \text{if the server is busy at time } t \end{cases}$$

So that the supplementary variables $B^0(t)$ and $V^0(t)$ are introduced in order to obtain a bivariate Morkov Process $\{N_0(t).L(t)\}$, where

$$L(t) = \begin{cases} V^0(t) & \text{if } Y(t) = 0 \\ B^0(t) & \text{if } Y(t) = 1 \end{cases}$$

We define-

$$P_{0,m}(X)dX = \lim_{n\to\infty} \Pr[N_Q(t)=m, L(t)=V^0 t;\ X < V^0(t) \le X+dX];\ \ X>0, m\ge 0$$

and $$P_{1,n}(X)dX = \lim_{n\to\infty} \Pr[N_Q(t)=n, L(t)=B^0(t);\ X < B^0(t) \le X+dX];\ X>0, n\ge 1$$

Now, the analysis of this queueing process at the stationary point of time can be done by using forward Kolmogorov equations, which under the steady state conditions can be written as:

$$\left(\frac{d}{dx}\right)P_{0,0}(X)+[\lambda+V(X)]P_{0,0}(X)=0; \qquad X>0 \quad ...(1)$$

$$\left(\frac{d}{dx}\right)P_{0,0}(X)+[\lambda+V(x)]P_{0,0}(X)=\lambda\sum_{k=1}^{n}C_k P_{0,\,n-k}(X); \quad X>0, n\ge 1 \quad ...(2)$$

$$\left(\frac{d}{dx}\right)P_{1,\,n}(X)+[\lambda+b(x)]P_{1,\,n}(X)=\lambda\sum_{k=1}^{n}C_k P_{1,\,n-k+1}(X); \quad X>0, n\ge 1 \quad ...(3)$$

$$\lambda P_{0,0} = \int_0^\infty V(X)P_{0,0}(X)dX + \int_0^\infty b(X)P_{1,1},(X)dX \quad ...(4)$$

Where,

$$P_{0,0} = \int_n^\infty P_{0,0}(X)dX$$

These conditions are to be solved under the following boundary conditions at X=0.

$$P_{0,\,0}(0)=\lambda P_{0,\,0} \quad ...(5)$$

$$P_{0,\,n}(0)=0; \qquad n\ge 1 \quad ...(6)$$

$$P_{1,\,n}(0)=\int_0^\infty V(X)P_{0,\,n}(X)dX+\int_0^\infty b(X)P_{1n+1}(X)dX; \quad n\ge 1 \quad ...(7)$$

and the normalization condition

$$\sum_{n=0}^{\infty}\int_0^\infty P_{0,1}(X)dX+\sum_{n=1}^{\infty}\int_0^\infty P_{1,\,n}(X)dX=1 \quad ...(8)$$

Let us define the following probability generating functions.

$$P_0(X,z)=\sum_{n=0}^{\infty}z^n P_{0n}(X); \qquad |z|\le 1$$

$$P_0(0,z)=\sum_{n=0}^{\infty}z^n P_{0n}(0) \qquad |z|\le 1$$

$$P_1(X,z)=\sum_{n=1}^{\infty}z^n P_{1,\,n}(X) \qquad |z|\le 1$$

$$P_1(0,z)=\sum_{n=1}^{\infty}z^n P_{1,\,n}(0) \qquad |z|\le 1$$

Proceeding in the usual manner with equations (1), (2), (3), (5) and (6), we get

$$P_0(X,z)=P_0(0;z)[1-V(X)]e^{-\lambda[1-X(z)]X}$$
$$=\lambda P_{0,\,0}[1-V(X)]e^{-\lambda[1-X(z)]X} \qquad x>0 \quad ...(9)$$

and

$$P_1(X;z)=P_1(0;z)[1-B(X)]e^{-\lambda[1-X(z)]X} \quad ...(10)$$

Thus, we have

$$P_0(z)=\int_0^\infty P_0(X,z)dX = P_{0,\,0}[1-V^*(b-\lambda X(z))]/[1-X(z)]$$

Similarly from equation (7) and (4), we have

$$P_1(0,z) = \lambda z P_{0,0}[1 - V^*(\lambda - \lambda X(z))]/[B^*(\lambda - \lambda X(z)) - z]$$

And therefore we get

$$P_1(z) = \int_0^\infty P_1(X,z)dX = \frac{z.P_{0,0}[1 - V^*(\lambda - \lambda X(z))][1 - B^*(\lambda - \lambda X(z))]}{B^*[(\lambda - \lambda X(z)) - z]}$$

Using the condition (8) and taking the limit of $[P_0(z) + P_1(z)]$, as $z \to 1$, we get

$$P_{0,0} = \frac{1-\rho}{\lambda E(V)}$$

Where $\rho = \alpha E(X)\ (<1)$ is the utilization factor of the system and $\alpha = \lambda E(B)\ (<1)$.

Let $P(z) = P_0(z) + P_1(z)$ be the probability generating function of the stationary queue size distribution of this M*/G/1 multiple vacation model, then

$$P(z) = \left[\frac{(1-\rho)(1-z)B^*[(\lambda - \lambda X(z))]}{B^*[(\lambda - \lambda X(z)) - z]}\right]\left[\frac{1 - V^*[(\lambda - \lambda X(z))]}{E(V)(\lambda - \lambda X(z))}\right] = P(\mathrm{M^*/G/l;\ z})S(z) \qquad ...(11)$$

where P(M*/G/1; Z) is the of the stationary point queue size distribution of the standard M*/G/1 queue without vacation. This is well known Pollaczek-Khinchine formula for M*/G/1 queue

$$= \left[\frac{(1-\rho)(1-z)B^*[(\lambda - \lambda X(z))]}{B^*[(\lambda - \lambda X(z)) - z]}\right]$$

and is the of the number of units that arrive during the residual life of the vacation time.

$$= \left[\frac{1 - V^*[(\lambda - \lambda X(z))]}{E(V)(\lambda - \lambda X(z))}\right]$$

Differentiating equation (11) with respect to and taking the limit as $z \to 1$, we get

$$L_S = \left[\frac{dP(z)}{dz}\right]_{z=1}$$

$$= \lambda E(x)E(V_R) + \rho + [\lambda^2 E(B^2)E^2(x) + \lambda E(B)E(X^2 - X)/2(1-\rho)]$$

where $E(V_R) = E(V^2)/2E(V)$ is the mean residual vacation time and L_s is the mean of the stationary queue size distribution of this model.

CONCLUSION

From equation 11 we observe that the stationary queue size distribution of M*/G/1 queue with multiple vacation is the convolution of distributions of two independent random variables on of which is the stationary queue size of the standard M*/G/1 queue without vacation and the other one is the number of units that arrive during the residual vacation time.

REFERENCES

1. Agarwal N.N. (1965): Some Proble in the Theory of Reliability and Queues, Ph.D. Thesis, Kurukshetra University, Kurukshetra.
2. Akyildiz I.F. (1988): On the exact and approximate through put analyis of closed queueing network with blocking, IEEE Trans. Software Eng. Se-14: 62-70.
3. Atitok T. (1989): Approximate analysis of queues in series with phase type service times and blocking, O.R., 27: 601-610.
4. Avi-llzak B. and Yadin M. (1965): A sequence of two servers with no intermediate queue, Mgmt.Sci., 11: 553-564.
5. Baba Y. (1986): On the M*/G/1 Queue with Vacation Time. Operations research Letters, 5: 93-98.

6. Baskett F., Chandy K.M.,Muntz R.R. and Palacious F.G. (1975): Open, closed and mixture networks of queues with different classes of customers, J. Assoc. Comput Mach., 22: 248-260.
7. Bombos N. and Wasserman K. (1994): On stationary tandem queueing networks with job feedback, Queueing Systems, 15: 137-164.
8. Bondi A.B. and Whitt W. (1986): The influence of service-time variablility in a closed network of queues. Perf. Eval., 6: 219-234.
9. Borthakur A. and Chaudhary G. (1997): On a Batch Arrival Poisson Process queue With Generalized Vacation, Sankhya Ser. B, 59: 369-382.
10. Boxma O. J., Kelly F.P. and Konheim A.G. (1984): The product form for sojourn time distributions in cyclic exponetial queues, J. Assoc.Comput, Mach., 31: 128-133.
11. EzhovI. and Kadankov V.F.(2002): Queueing Systems G*/G-1 with Customers Arriving in Batches. Theo. Prob. nd Math. Statist. No. 64:19-36.
12. Fuhrmann S.W. and Cooper R.B. (1985): Stochastic DecompsitionIn M/G/1 Queue with Generalized Vacation Operations Research, 33: 1117-1129.
13. Gross D. and Harris C.M. (1985): Fundamentals of Queueing Theory, 2 Edition John Wiley and Sons, New York.
14. Harris C.M. and Marchal W.G. (1988): State dependence in M/G/1 Server Vacation Models, Operations Research, 36: 560-565.
15. Hemker J. (1990): A note on sojourn times in queueing networks with multi-erver nodes,J. Appl. Prob., 27: 469-474.
16. Henderson W. and Taylor P.G. (1990): Open networks of queues with batch arrivals and batch services, Queueing Systems, 6: 71-88.