



RESEARCH ARTICLE

Stochastic Analysis of a System with Two Types of Failure and Preventive Maintenance

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Received: 1st May 2015, Revised: 15th June 2015, Accepted: 18th June 2015

ABSTRACT

The present paper deals with the Stochastic Analysis of a System with Two Types of Failure and Preventive Maintenance”, an attempt has been made to analyse a two identical units system. Upon failure of an operative unit, the cold standby unit becomes operative automatically by the help of a switch which is always perfect. The failure of operative unit occurs with two types of faults known as minor and major. The system is having two types of repair facilities i.e. ordinary and expert repairman. An operative unit is sent for preventive maintenance after continuously working for a fixed amount of time provided both the unit of system are alive so that the system cannot be in down position. The preventive maintenance of a unit will automatically stop whenever the other unit under operation fails. Whenever the standby unit is not alive the failure rates of an operative unit increase automatically because preventive maintenance cannot be possible in such a situation.

Key words: Stochastic Analysis, Failure and Preventive Maintenance

INTRODUCTION

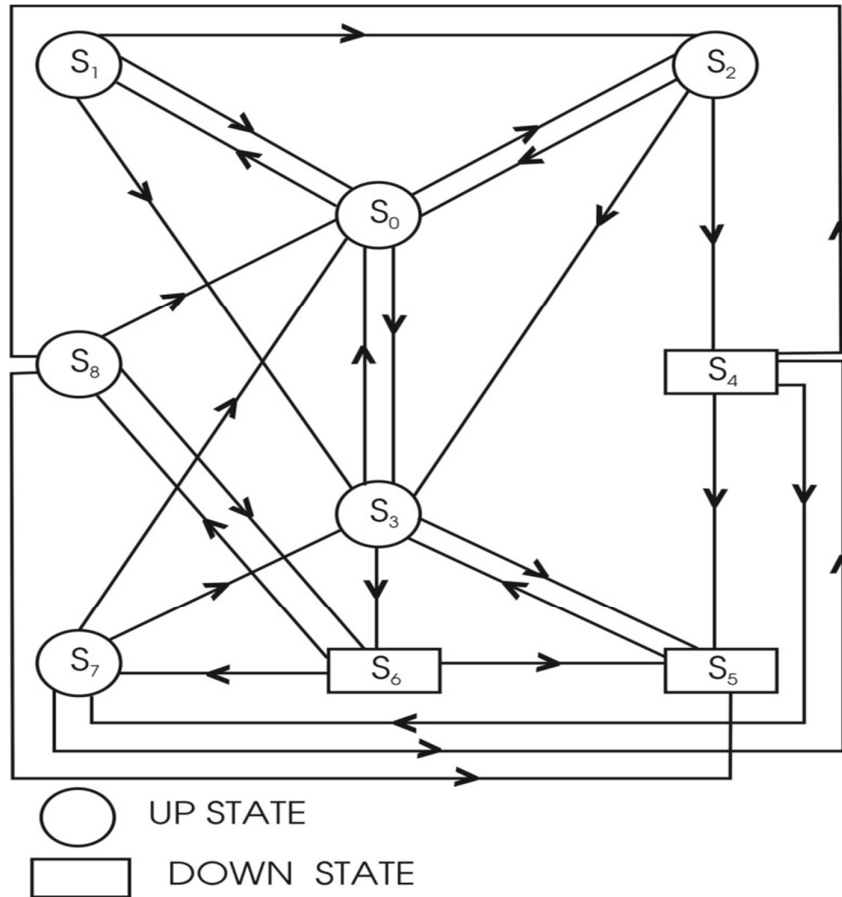
Workings in the field of reliability have analyzed several engineering systems by using different sets of assumptions. Most of them considered only a single repair facility to remove all type of faults. Also the operative unit operates continuously till it fails without sending it for preventive maintenance. But in the real practical situations it will be better to consider two repair facilities i.e. one is for minor faults and the other for major. Also for increasing the reliability of the system it is quite reasonable to provide preventive maintenance to the operative unit after a fixed amount of continuous operation. Keeping the above view, we in this chapter analyzed a repair facilities, preventive maintenance and adjustable failure rate of operative unit. Using regenerative point technique with Markov renewal process, the following reliability characteristics of interest are obtained.

MODEL DESCRIPTION AND ASSUMPTIONS

1. The system comprises of two units which are identical. Initially, one unit operative and the other is kept as cold standby.
2. Upon failure of an operative unit, the cold standby unit becomes operative automatically by the help of a switch which is always perfect.
3. The failure of operative unit occurs with two types of faults known as minor and major. Probability that an operative unit fails due to minor or major faults is fixed.
4. The system is having two types of repair facilities i.e. ordinary and expert repairman. The minor and major faults will be attended by ordinary and expert repairman respectively.
5. The time for repairing minor repair by ordinary repair facility is fixed and if he is unable to repair the failed unit within this time then expert repairman will be called to repair the failed unit. Probability that the ordinary repairman will complete the repair (minor) during this fixed time is fixed.
6. An operative unit is sent for preventive maintenance after continuously working for a fixed amount of time provided both the unit of system are alive so that the system cannot be in

- down position. Preventive maintenance will be performed by ordinary repairman and during this process, operative unit will be in down position.
7. The preventive maintenance of a unit will automatically stop whenever the other unit under operation fails.
 8. Whenever the standby unit is not alive the failure rate of an operative unit increases automatically because preventive maintenance cannot be possible in such a situation.
 9. Failure time distribution of operative unit is exponential.
 10. The time for completing all the jobs performed by ordinary and expert repairman follows exponential.

NOTATION AND SYMBOLS



- N_0 : Normal unit kept as operative
- N_{CS} : Normal unit kept as cold standby
- N_{PM} : Normal unit under preventive maintenance
- F_{or} : Failed unit under repair by ordinary repairman
- F_{er} : Failed unit under repair by expert repairman
- F_{wer} : Failed unit waiting for expert repairman
- F_{OR} : Repair of the failed unit by ordinary repairman is continued from earlier state
- F_{ER} : Repair of the failed unit by expert repairman is continued from earlier state
- α : Failure rate of an operative unit when the other unit is alive
- $\beta(>\alpha)$: Failure rate of an operative unit when the other unit is not alive
- γ : Rate of completing repair of failed unit by ordinary repairman
- δ : Rate of completing repair of failed unit by expert repairman

- ϕ : Rate of completing preventive maintenance of an operative unit
 P : Probability that failure of the operative unit occur due to minor fault
 Q : Probability that failure of the operative unit occur due to major fault
 p_1 : Probability that ordinary repairman will complete the repair of failed unit within a fixed amount of time
 q_1 : Probability that ordinary repairman will not complete the repair of failed unit within a fixed amount of time

Using the above notation and symbols the possible states of the system are-

Up States:

$$S_0 \equiv (N_0, N_{CS}) \quad S_1 \equiv (N_0, N_{PM}) \quad S_2 \equiv (N_0, F_{or})$$

$$S_3 \equiv (N_0, F_{er}) \quad S_7 \equiv (N_0, F_{or}) \quad S_8 \equiv (N_0, F_{ER})$$

Down States:

$$S_4 \equiv (F_{OR}, F_{er}) \quad S_5 \equiv (F_{wer}, F_{ER}) \quad S_6 \equiv (F_{OR}, F_{ER})$$

The transitions between the various states are shown in Fig.

TRANSITION PROBABILITIES

Let $T_0(=0), T_1, T_2$ denotes the entry into any state $S_i \in E$. Let X_n be the states visited at epoch T_{n+1} i.e. just after the transition at T_n . Then $\{T_n, X_n\}$ is a Markov-renewal process with state space E and is semi Markov-Kernel over E .

$$Q_{ij}(t) = \Pr[X_{n+1} = S_j, T_{n+1} - T_n \leq t \mid X_n = S_i] \quad (1)$$

The stochastic matrix of the embedded Markov chain is-

$$P = (p_{ij}) = Q_{ij}(\infty) = Q(\infty). \quad (2)$$

(2)

the non-zero elements of P_{ij} are given below:

$$p_{01} = \frac{\phi}{\alpha + \phi} \quad p_{02} = p \cdot \frac{\alpha}{\alpha + \phi}$$

$$p_{03} = q \cdot \frac{\alpha}{\alpha + \phi} \quad p_{10} = \frac{\gamma}{\alpha + \gamma}$$

$$p_{12} = p \cdot \frac{\alpha}{\alpha + \gamma} \quad p_{13} = q \cdot \frac{\alpha}{\alpha + \gamma}$$

$$p_{20} = p_1 \cdot \frac{\gamma}{\beta + \gamma} \quad p_{23} = q_1 \cdot \frac{\gamma}{\beta + \gamma}$$

$$p_{24} = \frac{\beta}{\beta + \gamma} \quad p_{30} = \frac{\delta}{\beta + \delta}$$

$$p_{35} = q \cdot \frac{\beta}{\beta + \delta} \quad p_{36} = p \cdot \frac{\beta}{\beta + \delta}$$

$$p_{45} = q_1 \cdot \frac{\gamma}{\delta + \gamma} \quad p_{47} = \frac{\delta}{\delta + \gamma}$$

$$p_{48} = p_1 \cdot \frac{\gamma}{\delta + \gamma} \quad p_{53} = 1$$

$$p_{65} = q_1 \cdot \frac{\gamma}{\delta + \gamma} \quad p_{67} = \frac{\delta}{\delta + \gamma}$$

$$p_{68} = p_1 \cdot \frac{\gamma}{\delta + \gamma} \quad p_{70} = p_1 \cdot \frac{\gamma}{\beta + \gamma}$$

$$\begin{aligned}
 p_{73} &= q_1 \cdot \frac{\gamma}{\beta + \gamma} & p_{74} &= \frac{\beta}{\beta + \gamma} \\
 p_{80} &= \frac{\delta}{\beta + \delta} & p_{85} &= q_1 \cdot \frac{\beta}{\beta + \delta} \\
 p_{86} &= p \cdot \frac{\beta}{\beta + \delta} & &
 \end{aligned}
 \tag{3-27}$$

27)

From the above probabilities the following relation can be easily verifies as-

$$\begin{aligned}
 p_{01} + p_{02} + p_{03} &= 1 & p_{10} + p_{12} + p_{135} &= 1 \\
 p_{20} + p_{23} + p_{24} &= 1 & p_{30} + p_{35} + p_{36} &= 1 \\
 p_{45} + p_{47} + p_{48} &= 1 & p_{53} &= 1 \\
 p_{65} + p_{67} + p_{68} &= 1 & p_{70} + p_{73} + p_{74} &= 1 \\
 p_{80} + p_{85} + p_{86} &= 1 & &
 \end{aligned}
 \tag{28-36)$$

MEAN SOJOURN TIMES

The mean sojourn time in a state S_i is defined as the length of stay in time in a state S_i before transiting to any other state.

If T denotes the sojourn time in S_i , then

$$\mu_i = E(T) = \int_0^{\infty} P_i[T > t] dt$$

(37)

Using this we can obtain the following expressions

$$\begin{aligned}
 \mu_0 &= \frac{1}{\alpha + \phi} & \mu_1 &= \frac{1}{\alpha + \gamma} & \mu_2 &= \frac{1}{\beta + \gamma} \\
 \mu_3 &= \frac{1}{\beta + \delta} & \mu_4 &= \frac{1}{\delta + \gamma} & \mu_5 &= \frac{1}{\delta} \\
 \mu_6 &= \frac{1}{\delta + \gamma} & \mu_7 &= \frac{1}{\beta + \gamma} & \mu_8 &= \frac{1}{\beta + \delta}
 \end{aligned}
 \tag{38-46)$$

MEAN TIME TO SYSTEM FAILURE (MTSF)

To obtain the distribution function $\pi_i(t)$ of the time to system failure with starting state S_0 .

$$\begin{aligned}
 \pi_0(t) &= Q_{01}(t)\pi_1(t) + Q_{02}(t)\pi_2(t) + Q_{03}(t)\pi_3(t) \\
 \pi_1(t) &= Q_{10}(t)\pi_0(t) + Q_{12}(t)\pi_2(t) + Q_{13}(t)\pi_3(t) \\
 \pi_2(t) &= Q_{20}(t)\pi_0(t) + Q_{23}(t)\pi_3(t) + Q_{24}(t) \\
 \pi_3(t) &= Q_{30}(t)\pi_0(t) + Q_{35}(t) + Q_{36}(t) \\
 \pi_7(t) &= Q_{70}(t)\pi_0(t) + Q_{73}(t)\pi_3(t) + Q_{74}(t) \\
 \pi_8(t) &= Q_{80}(t)\pi_0(t) + Q_{85}(t) + Q_{86}(t)
 \end{aligned}
 \tag{47-52)$$

Taking Laplace Stieltjes transform of relations and solving for $\tilde{\pi}_0(s)$, we get;

$$\tilde{\pi}_0(s) = N_1(s) / D_1(s)$$

(53)

where

$$N_1(s) = \tilde{Q}_{24}(\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_{02}) + (\tilde{Q}_{35} + \tilde{Q}_{36})\{\tilde{Q}_{01}(\tilde{Q}_{12}\tilde{Q}_{23} + \tilde{Q}_{13}) + \tilde{Q}_{03} + \tilde{Q}_{02}\tilde{Q}_{23}\}$$

(54)

and

$$D_1(s) = 1 - \tilde{Q}_{01}\tilde{Q}_{10} - \tilde{Q}_{20}(\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_{02}) - \tilde{Q}_{30}\{\tilde{Q}_{01}(\tilde{Q}_{12}\tilde{Q}_{23} + \tilde{Q}_{13}) + \tilde{Q}_{03} + \tilde{Q}_{02}\tilde{Q}_{23}\}$$

(55)

By taking the limit $s \rightarrow 0$ in equation (53), one gets $\tilde{\pi}_0(0) = 1$, which implies that $\tilde{\pi}_0(t)$ is a proper distribution function.

$$E(T) = - \frac{d}{ds} \pi_0(s)|_{s=0} = \frac{D'_1(0) - N'_1(0)}{D_1(0)} = N_1/D_1$$

(56)

where

$$N_1 = \mu_0 + p_{01}\mu_1 + (p_{01}p_{12} + p_{02})\mu_2 + \mu_3\{p_{01}(p_{12}p_{23} + p_{13}) + p_{03} + p_{02}p_{23}\}$$

(57)

and

$$D_1 = 1 - p_{01}p_{10} - p_{20}(p_{01}p_{12} + p_{02}) - p_{30}\{p_{01}(p_{12}p_{23} + p_{13}) + p_{03} + p_{02}p_{23}\}$$

(58)

AVAILABILITY ANALYSIS

System availability is defined as,

$A_i(t) = \Pr$ [Starting from state S_i the system is available at epoch t without passing through any regenerative state]

and $M_i(t) = \Pr$ [Starting from upstate S_i the system remains up till epoch without passing through any regenerative up state]

Thus,

$$\begin{aligned} M_0(t) &= e^{-(\alpha+\phi)t} & M_1(t) &= e^{-(\alpha+\gamma)t} & M_2(t) &= e^{-(\beta+\gamma)t} \\ M_3(t) &= e^{-(\beta+\delta)t} & M_7(t) &= e^{-(\beta+\gamma)t} & M_8(t) &= e^{-(\beta+\delta)t} \end{aligned}$$

(59-64)

Now, obtaining $A_i(t)$ by using elementary probability argument;

$$A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) + q_{03}(t) \odot A_3(t)$$

$$A_1(t) = M_1(t) + q_{10}(t) \odot A_0(t) + q_{12}(t) \odot A_2(t) + q_{13}(t) \odot A_3(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) \odot A_0(t) + q_{23}(t) \odot A_3(t) + q_{24}(t) \odot A_4(t)$$

$$A_3(t) = M_3(t) + q_{30}(t) \odot A_0(t) + q_{35}(t) \odot A_5(t) + q_{36}(t) \odot A_6(t)$$

$$A_4(t) = q_{45}(t) \odot A_5(t) + q_{47}(t) \odot A_7(t) + q_{48}(t) \odot A_8(t)$$

$$A_5(t) = q_{53}(t) \odot A_3(t)$$

$$A_6(t) = q_{65}(t) \odot A_5(t) + q_{67}(t) \odot A_7(t) + q_{68}(t) \odot A_8(t)$$

$$A_7(t) = M_7(t) + q_{70}(t) \odot A_0(t) + q_{73}(t) \odot A_3(t) + q_{74}(t) \odot A_4(t)$$

$$A_8(t) = M_8(t) + q_{80}(t) \odot A_0(t) + q_{85}(t) \odot A_5(t) + q_{86}(t) \odot A_6(t)$$

(65-73)

Now taking Laplace transform of above equations (65-73) for pointwise availability $A^*_0(s)$, we get;

$$A^*_0(s) = \frac{N_2(s)}{D_2(s)}$$

(74)

Where in terms of

$$\begin{aligned}
 a &= q^*_{01}q^*_{12} + q^*_{02}, \\
 b &= q^*_{01}(q^*_{12}q^*_{23} + q^*_{13}) + (q^*_{03} + q^*_{02}q^*_{23}) \\
 c &= q^*_{48} + q^*_{45}q^*_{53} + q^*_{23}q^*_{47}
 \end{aligned} \tag{75-77}$$

We have,

$$\begin{aligned}
 N_2(s) &= [(1 - q^*_{24}q^*_{47})(1 - q^*_{35}q^*_{53}) - q^*_{36}.c] (M^*_{0} + q^*_{01}M^*_{1} + M^*_{2}.a) \\
 &\quad + [M^*_{3}(1 - q^*_{24}q^*_{47}) + q^*_{36}q^*_{47}M^*_{2}].b \\
 &\quad + q^*_{24}[(1 - q^*_{35}q^*_{53})q^*_{47}M^*_{2} + M^*_{3}.c].a
 \end{aligned} \tag{78}$$

And

$$\begin{aligned}
 D_2(s) &= [(1 - q^*_{24}q^*_{47})(1 - q^*_{35}q^*_{53}) - q^*_{36}.c](1 - q^*_{01}q^*_{10} - q^*_{20}.a) \\
 &\quad - [q^*_{30}(q^*_{01}(1 - q^*_{24}q^*_{47}) + q^*_{36}q^*_{20}q^*_{47}).b \\
 &\quad - q^*_{24}[(1 - q^*_{35}q^*_{53})q^*_{20}q^*_{47} + c]].a
 \end{aligned} \tag{79}$$

By taking the limit $s \rightarrow 0$ in the relation (79), one gets the value of $D_2(0) = 0$, therefore the steady state availability of the system when it starts operations from S_0 is

$$A_0(\infty) = \lim_{t \rightarrow \infty} A_0(t) = \lim_{s \rightarrow 0} s \cdot A_0^*(s) = N_2(0)/D'_2(0) = N_2/D_2 \tag{80}$$

Where

$$\begin{aligned}
 N_2 &= [(1 - p_{24}p_{47})p_{30} + p_{36}p_{47}p_{20}].[\mu_0 + p_{01}\mu_1 + \mu_2(p_{01}p_{12} + p_{02})] \\
 &\quad [\mu_3(1 - p_{24}p_{47}) + p_{36}p_{47}\mu_2].[p_{01}(p_{12}p_{23} + p_{13}) + (p_{03} + p_{02}p_{23})] \\
 &\quad + p_{24}[(1 - p_{35})p_{47}\mu_2 + \mu_3(p_{48} + p_{45} + p_{23}p_{47})](p_{01}p_{12} + p_{02})
 \end{aligned} \tag{81}$$

and

$$\begin{aligned}
 D_2 &= (\mu_0 + p_{01}\mu_1)[(1 - p_{24}p_{47})p_{30} + p_{36}p_{47}p_{20}] + (1 - p_{01}p_{10}) [p_{36}(p_{47}\mu_2 \\
 &\quad + \mu_4 + p_{45}\mu_5) + (1 - p_{24}p_{47})(\mu_3 + p_{35}\mu_5)] + (p_{01}p_{12} + p_{02})[p_{30}\mu_2 \\
 &\quad - p_{20}(\mu_3 + p_{35}\mu_5) - (\mu_4 + p_{45}\mu_5)(p_{20}p_{36} + p_{24}p_{30})]
 \end{aligned} \tag{82}$$

BUSY PERIOD ANALYSIS

Let us define $W_i(t)$ as the probability that the system is under repair by repair facility in state $S_i \in E$ at time t without transiting to any regenerative state. Therefore

$$\begin{aligned}
 W_1(t) &= e^{-(\alpha+\gamma)t} & W_2(t) &= e^{-(\beta+\gamma)t} & W_3(t) &= e^{-(\beta+\delta)t} \\
 W_4(t) &= e^{-(\delta+\gamma)t} & W_5(t) &= e^{-\delta t} & W_6(t) &= e^{-(\delta+\gamma)t} \\
 W_7(t) &= e^{-(\beta+\gamma)t} & W_8(t) &= e^{-(\beta+\delta)t}
 \end{aligned} \tag{83-90}$$

Also let $B_i(t)$ is the probability that the system is under repair by repair facility at time t , Thus the following recursive relations among $B_i(t)$'s can be obtained as ;

$$\begin{aligned}
 B_0(t) &= q_{01}(t) \odot B_1(t) + q_{02}(t) \odot B_2(t) + q_{03}(t) \odot B_3(t) \\
 B_1(t) &= W_1(t) + q_{10}(t) \odot B_0(t) + q_{12}(t) \odot B_2(t) + q_{13}(t) \odot B_3(t) \\
 B_2(t) &= W_2(t) + q_{20}(t) \odot B_0(t) + q_{23}(t) \odot B_3(t) + q_{24}(t) \odot B_4(t) \\
 B_3(t) &= W_3(t) + q_{30}(t) \odot B_0(t) + q_{35}(t) \odot B_5(t) + q_{36}(t) \odot B_6(t) \\
 B_4(t) &= W_4(t) + q_{45}(t) \odot B_5(t) + q_{47}(t) \odot B_7(t) + q_{48}(t) \odot B_8(t) \\
 B_5(t) &= W_5(t)q_{53}(t) \odot B_3(t) \\
 B_6(t) &= W_6(t) + q_{65}(t) \odot B_5(t) + q_{67}(t) \odot B_7(t) + q_{68}(t) \odot B_8(t) \\
 B_7(t) &= W_7(t) + q_{70}(t) \odot B_0(t) + q_{73}(t) \odot B_3(t) + q_{74}(t) \odot B_4(t) \\
 B_8(t) &= W_8(t) + q_{80}(t) \odot B_0(t) + q_{85}(t) \odot B_5(t) + q_{86}(t) \odot B_6(t)
 \end{aligned} \tag{91-99}$$

Taking Laplace transform of the equations (91-99) and solving for $B^*_0(s)$, we get;

$$B^*_0(s) = N_3(s)/D_3(s) \tag{100}$$

Where $D_3(s)$ is same as $D_2(s)$ in (79) and

$$N_3(s) = [(1 - q^{*24}q^{*47})(1 - q^{*35}q^{*53}) - q^{*36}.c] (q^{*01}W^{*1} + W^{*2}.a) + [W^{*3}(1 - q^{*24}q^{*47}) + q^{*36}q^{*47}W^{*2}].b + q^{*24}[(1 - q^{*35}q^{*53})q^{*47}W^{*2} + W^{*3}.c].a + W^{*4}[a.\{q^{*23}q^{*36} + q^{*24}(1 - q^{*35}q^{*53})\} + (q^{*01}q^{*13} + q^{*03})q^{*36}] + W^{*5}[a.\{q^{*35}(q^{*23} + q^{*24}q^{*48}) + q^{*45}(q^{*23}q^{*36} + q^{*24})\} + (q^{*01}q^{*13} + q^{*03}).\{q^{*35}(1 - q^{*24}q^{*47}) + q^{*45}q^{*36}\}] \quad (101)$$

Where a, b and c are same as in (75-77).

In this steady state, the fraction of time for which the repair facility is busy in repair is given by

$$B_0 = \lim_{t \rightarrow \infty} B_0(t) = \lim_{s \rightarrow 0} s B^*(s) = N_3(0)/D'_3(0) = N_3/D_3 \quad (102)$$

where D_3 is same as D_2 in (82) and

$$N_3 = [(1 - p_{24}p_{47})p_{30} + p_{36}p_{47}p_{20}].[\mu_0 + p_{01}\mu_1 + \mu_2(p_{01}p_{12} + p_{02})] + [\mu_3(1 - p_{24}p_{47}) + p_{36}p_{47}\mu_2].\{p_{01}(p_{12}p_{23} + p_{13}) + (p_{03} + p_{02}p_{23})\} + p_{24}[(1 - p_{35})p_{47}\mu_2 + \mu_3(p_{48} + p_{45} + p_{23}p_{47})](p_{01}p_{12} + p_{02}) + \mu_4[(p_{01}p_{12} + p_{02})\{(1 - p_{20})p_{36} + p_{24}p_{30}\} + (p_{01}p_{13} + p_{03})p_{36}] + \mu_5[(p_{01}p_{12} + p_{02}).\{p_{35}(p_{23} + p_{24}p_{48}) + p_{45}(p_{23}p_{36} + p_{24})\} + (p_{01}p_{13} + p_{03}).\{p_{35}(1 - p_{24}p_{47}) + p_{45}p_{36}\}] \quad (103)$$

EXPECTED NUMBER OF VISITS BY THE REPAIR FACILITY

Let we define, $V_i(t)$ as the expected number of visits by the repair facility in $(0,t]$ given that the system initially started from regenerative state S_i at $t=0$.

Then following recurrence relations among $V_i(t)$'s can be obtained as;

$$\begin{aligned} V_0(t) &= Q_{01}(t)[1 + V_1(t)] + Q_{02}(t)[1 + V_2(t)] + Q_{03}(t)[1 + V_3(t)] \\ V_1(t) &= Q_{10}(t)V_0(t) + Q_{12}(t)V_2(t) + Q_{13}(t)V_3(t) \\ V_2(t) &= Q_{20}(t)V_0(t) + Q_{23}(t)V_3(t) + Q_{24}(t)V_4(t) \\ V_3(t) &= Q_{30}(t)V_0(t) + Q_{35}(t)V_5(t) + Q_{36}(t)V_6(t) \\ V_4(t) &= Q_{45}(t)V_5(t) + Q_{47}(t)V_7(t) + Q_{48}(t)V_8(t) , \\ V_5(t) &= Q_{53}(t)V_3(t) \\ V_6(t) &= Q_{65}(t)V_5(t) + Q_{67}(t)V_7(t) + Q_{68}(t)V_8(t) \\ V_7(t) &= Q_{70}(t)V_0(t) + Q_{73}(t)V_3(t) + Q_{74}(t)V_4(t) \\ V_8(t) &= Q_{80}(t)V_0(t) + Q_{85}(t)V_5(t) + Q_{86}(t)V_6(t) \end{aligned} \quad (104-112)$$

Taking Laplace stieltjes transform of the above equations and solving for $\tilde{V}_0(s)$, we get;

$$\tilde{V}_0(s) = N_4(s)/D_4(s) \quad (113)$$

where in terms of

$$A = \tilde{Q}_{01} \tilde{Q}_{12} + \tilde{Q}_{02}$$

$$B = \tilde{Q}_{01}(\tilde{Q}_{12} \tilde{Q}_{23} + \tilde{Q}_{13}) + (\tilde{Q}_{03} + \tilde{Q}_{02} \tilde{Q}_{23})$$

$$C = \tilde{Q}_{48} + \tilde{Q}_{45} \tilde{Q}_{53} + \tilde{Q}_{23} \tilde{Q}_{47} \quad (114-116)$$

We get

$$N_4(s) = [(1 - \tilde{Q}_{24} \tilde{Q}_{47})(1 - \tilde{Q}_{35} \tilde{Q}_{53}) - \tilde{Q}_{36}.C](\tilde{Q}_{01} + \tilde{Q}_{02} + \tilde{Q}_{03}) \quad (117)$$

And

$$D_4(s) = [(1 - \tilde{Q}_{24} \tilde{Q}_{47})(1 - \tilde{Q}_{35} \tilde{Q}_{53}) - \tilde{Q}_{36}.C](1 - \tilde{Q}_{01} \tilde{Q}_{10} - \tilde{Q}_{20}.A)$$

$$[\tilde{Q}_{30}(1 - \tilde{Q}_{24} \tilde{Q}_{47}) + \tilde{Q}_{36} \tilde{Q}_{20} \tilde{Q}_{47}].B$$

$$- \tilde{Q}_{24}[(1 - \tilde{Q}_{35} \tilde{Q}_{53}) \tilde{Q}_{20} \tilde{Q}_{47} + \tilde{Q}_{30}.C].A$$

(118)

In steady state the number of visit per unit of time when the system starts after entrance into state S_0 is;

$$V_0 = \lim_{t \rightarrow \infty} [V_0(t)/t] = \lim_{s \rightarrow 0} s \tilde{V}_0(s) = N_4/D_4 \quad (119)$$

$$t \rightarrow \infty \quad s \rightarrow 0$$

where D_4 is same as D_2 in (82) and

$$N_4 = (1 - p_{24}p_{47})p_{30} + p_{20}p_{36}p_{47} \quad (120)$$

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