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RESEARCH ARTICLE

MHD Free Convective Flow of Stratified Viscous Fluid through Porous Media with Hall Currents

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ABSTRACT

In this section, we study of unsteady MHD free convection of an incompressible, electrically conducting, viscous, stratified liquid through a porous medium past a vertical, porous, isothermal infinite plate fluctuating with hall currents and time dependent suction velocity is presented. The flow is considered under the influence of transversely applied uniform magnetic field. It is assumed that the magnetic Reynolds number is very small so that the induced magnetic field is negligible. To obtain the solution for the velocity field and temperature field of the problem. The skin-friction coefficients at the plat and heat transfer in terms of Nusselt number are also obtained. The results obtained are discussed graphically and numerically.
Key words: *Magnetic field, stratified viscous fluid, porous medium, heat transfer, Hall current*

INTRODUCTION

The solution of fluid flow with different flow conditions and the corresponding energy equation with different wall temperature including heat generated due to friction etc. is well reported in literature. At present fluid flow through porous media is of considerable importance in a wide range of disciplines in science and technology viz., in soil mechanics, ground water, hydrology, petroleum engineering, water purification, industrial filtration, ceramic engineering and power metallurgy. The importance has been recognized in agricultural engineering to study the underground water resources, seepage of water in river beds, in chemical engineering for filtration and purification process, in petroleum technology to study the movements of natural gas, oil and water through the oil reservoirs. The theory of laminar flow through homogeneous porous media is based on an experiment originally conducted by Darcy. Ahmadi & Manvi (1971) gave a general equation governing the motion of a viscous fluid through rigid porous medium and applied the results obtainedk2 to some basic flow problems.

The flow problems on convective flow have been studied by several authors. Gersten & Gross (1974), Yamamoto & Iwamura (1976), Ram & Mishra (1977), Singh & Singh (1989), Purushothaman, *et al.*, (1990), Singh & Rana (1992) and Singh, *et al.,* (1997) have studied convective flow problems under different physical situations and corresponding boundary conditions.

The study of flow through porous media with stratified flow is of considerable interest due to its application in soil sciences, petroleum industry, chemical engineering filtration processes and so on. The study of stratified fluid flows in presence of uniform magnetic field has its application in astrophysics, separation of oils of different densities, and several other application in engineering and technology. The study of effects of various physical variables on stratified viscous flow has been studied by so many authors including Singh & Singh (1991), Das and Nandy (1995), Ram *et. al.,* (1993), Soundalgekar and Uplekar (1995), Soundalgekar *et. al.,* (1996), Singh (1996), Singh *et. al.,* (1998) and several other authors. Recently, Pundhir *et. al.,* (2009) have discussed on hydromagnetic free convective flow of stratified viscous fluid past a porous vertical plate.

In this section our aim to investigate the effects of Hall current on unsteady hydromagnetic free convection of an incompressible, electrically conducting, viscous, stratified liquid in a porous medium past a vertical, porous, isothermal infinite plate fluctuating with time dependent suction velocity is presented. The flow is considered under the influence of transversely applied uniform magnetic field. It is assumed that the magnetic Reynolds number is very small so that the induced magnetic field is negligible. To obtain the solution for the velocity field and temperature field of the problems, the technique suggested by Lighthill (1954) is used. In addition skin-friction coefficients at the plate and heat transfer in terms of Nusselt number are also obtained. The results obtained are discussed graphically and numerically.

FORMULATION OF THE PROBLEM

We consider the unsteady convective flow of an incompressible, electrically conducting viscous, stratified fluid through a porous medium past a vertical, porous, isothermal infinite plate with time dependent suction at the plate under a magnetic applied normal to the flow. Let x-axis be in the plane of the plate along the direction of the flow and y-axis perpendicular to the plate and passes through the x-axis. We further assume that the density (ρ), viscosity (μ), thermal conductivity (K_T) and volumetric coefficient expansion (β') satisfy exponential law viz. $\rho = \rho_o \exp(-\beta y)$, $\mu = \mu_o \exp(-\beta y)$, $K_T = K_{T0} \exp(-\beta y)$ and $\beta' = \beta_0^{'} \exp(-\beta y)$

where β is the stratification factor and y is the distance perpendicular to the plate. In addition, the magnetic field $B = B_0 \exp\left(-\frac{\beta y}{2}\right)$ is applied perpendicular to the flow region and the suction velocity $v = v_0[1+\in f(t)]$ is a function of time and the pressure gradient is negligible. In addition to above considerations, the present analysis is made on the basis of following assumptions:

1. All the fluid properties are constant except the influence of the density variation with temperature only in body force term.
2. In the influence of density variation, other terms of the momentum equation and variation of expansion coefficient with temperature are considered negligible.
3. The free convection currents are in existence due to temperature difference ($T - T_\infty$).

Hence, the governing equation of motions and energy for the flow of fluid under the present configuration are:

Momentum Equation:

$$\rho\left[\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right] = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\mu}{k}u - \frac{\sigma B^2}{(1+m^2)}u + g\beta'(T - T_\infty) \quad ...(1)$$

Energy Equation:

$$\rho C_P\left[\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right] = K_T\frac{\partial^2 T}{\partial y^2} \quad ...(2)$$

Where,
u = the velocity in the direction of x-axis.
T = the temperature of the liquid.
T_∞ = the temperature of the fluid far from the plate.
β' = the volumetric coefficient of thermal expansion.
σ = the conductivity of the liquid.
K_T = the thermal conductivity.
m = the Hall current parameter.
C_P = the specific heat at constant pressure.

Introducing the values of ρ, μ, K_T, β' and $\vec{B}$, we obtain

$$\frac{\partial u}{\partial t}-v_0\left[1+\in f(t)\right]\frac{\partial u}{\partial y}=\upsilon_0\frac{\partial^2 u}{\partial y^2}-\upsilon_0\beta\frac{\partial u}{\partial y}-\frac{\upsilon_0}{K}u$$

$$-\frac{\sigma B_0^2}{(1+m^2)\rho_0}u+g\beta_0'\left(T-T_\infty\right) \qquad ...(3)$$

$$\frac{\partial T}{\partial t}-v_0\left[1+\in f(t)\right]\frac{\partial T}{\partial y}=\frac{K_{To}}{C_P}\frac{\partial^2 T}{\partial y^2} \qquad ...(4)$$

The boundary conditions for the present problem are

$$\left.\begin{array}{llll} u=v_0\left[1+\in f(t)\right], & T=T_w\left[1+\in f(t)\right] & at & y=0 \\ u\to 0, & T\to T_\infty & as & y\to\infty \end{array}\right\} \qquad ...(5)$$

We introduce the following non-dimensional variables:

$$u^*=\frac{u}{v_0},\quad y^*=\frac{yv_0}{\upsilon_0},\quad t^*=\frac{tv_0^2}{\upsilon_0^2},\quad k^*=\frac{kv_0^2}{\upsilon_0^2}\quad and\quad T^*=\frac{T-T_\infty}{T_w-T_\infty}$$

Introducing the above non-dimensional variables, the equations (3) and (4) reduce to

$$\frac{\partial u}{\partial t}-\left[1+\in f(t)\right]\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-S\frac{\partial u}{\partial y}-\left(\frac{M^2}{1+m^2}+\frac{1}{K}\right)u+G_rT \qquad ...(6)$$

$$\frac{\partial T}{\partial t}-\left[1+\in f(t)\right]\frac{\partial T}{\partial y}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} \qquad ...(7)$$

Where $G_r=\dfrac{g\beta_0{}'(Tw-T_\infty)\upsilon_0}{v_0^3}$ (Grashof number)

$S=\dfrac{\beta\upsilon_0}{v_0}$ (Stratification factor)

$M=\dfrac{\sigma B_0}{v_0}\sqrt{\dfrac{\sigma\upsilon}{\rho_0}}$ (Magnetic parameter)

And $P_r=\dfrac{\mu_0 C_p}{K_{T0}}$ (Prandtl number)

The boundary conditions (5) are transformed to

$$\left.\begin{array}{llll} u=1+\in f(t), & T=1+\in f(t) & at & y=0 \\ u\to 0, & T\to 0 & as & y\to\infty \end{array}\right\} \qquad ...(8)$$

SOLUTION OF THE PROBLEM

To obtain the solution of the problem, we consider the case when the suction velocity is exponentially decreasing function of the time. Hence we assume-

$$f(t)=e^{-nt} \qquad ...(9)$$

Substituting (9) in the equation (6) and (7), we get

$$\frac{\partial u}{\partial t}-\left[1+\in e^{-nt}\right]\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-S\frac{\partial u}{\partial y}-\left(\frac{M^2}{1+m^2}+\frac{1}{K}\right)u+G_rT \qquad ...(10)$$

$$\frac{\partial T}{\partial t}-\left[1+\in e^{-nt}\right]\frac{\partial T}{\partial y}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} \qquad ...(11)$$

The boundary conditions (8) are transformed to

$$\left.\begin{array}{llll} u=1+\in e^{-nt}, & T=1+\in e^{-nt} & at & y=0 \\ u\to 0, & T\to 0 & as & y\to\infty \end{array}\right\} \qquad ...(12)$$

Following Lightill (1954), we assume the velocity field and the temperature field as:

$$\left.\begin{array}{l} u(y,t)=u_0(y)+\in u_1(y)e^{-nt} \\ T(y,t)=T_0(y)+\in T_1(y)e^{-nt} \end{array}\right\} \qquad ...(13)$$

Substituting (13) in the equations (10) and (11) and equating the harmonic and non-harmonic terms, we get

$$u_0''(y)+(1-S)u_0'(y)-M_1u_0(y)=-G_rT_0 \qquad ...(14)$$

$$u_1''(y)+(1-S)u_1'(y)-(M_1-n)u_1(y)=-G_rT_1-u_1' \qquad ...(15)$$

$$T_0''(y)+P_rT_0'(y)=0 \qquad ...(16)$$

$$T_1''(y)+P_rT_1'(y)+nP_rT_1(y)=-P_rT_0' \qquad ...(17)$$

The boundary conditions (12) are transformed to

$$\left.\begin{array}{llllll} u_0=1, & u_1=1, & T_0=1, & T_1=1 & at & y=0 \\ u_0\to 0, & u_1\to 0, & T_0\to 0, & T_0\to 0 & as & y\to\infty \end{array}\right\} \qquad ...(18)$$

The solution of the above coupled equations, using the corresponding boundary conditions is obtained as:

$$T_0(y)=e^{-\Pr y} \qquad ...(19)$$

$$T_1(y)=\left(1-\frac{P_r}{n}\right)e^{-M_2y}+\frac{P_r}{n}e^{-P_ry} \qquad ...(20)$$

$$u_0(y)=(1+K_1)e^{-M_2y}-K_1e^{-P_ry} \qquad ...(21)$$

$$u_1(y)=(1+K_3+K_5+K_6)e^{-M_4y}-k_3e^{-M_2y}-K_5e^{-M_3y}-K_6e^{-P_ry} \qquad ...(22)$$

Where,

$$M_1=\frac{M^2}{1+m^2}+\frac{1}{K}$$

$$M_2=\frac{P_r+\sqrt{P_r\left(P_r+4n\right)}}{2}$$

$$M_3 = \frac{(1-S)+\sqrt{(1-S)^2+4M_1}}{2}$$

$$M_4 = \frac{(1-S)+\sqrt{(1-S)^2+4(M_1-n)}}{2}$$

$$K_1 = \frac{Gr}{P_r^2-(1-S)P_r-M_1}$$

$$K_3 = \frac{Gr\left(1-\frac{\Pr}{n}\right)}{M_2^2-(1-S)M_2-(M_1-n)}$$

$$K_5 = \frac{M_3(1-K_1)}{M_3^2-(1-S)M_3-(M_1-n)}$$

$$K_6 = \frac{\left(\frac{Gr\Pr}{n}+K_1\Pr\right)}{\Pr^2-(1-S)\Pr-(M_1-n)}$$

Substituting the value of $u_0(y)$, $u_1(y)$, $T_0(y)$ and $T_1(y)$ from (19) to (22) in (13), we obtain

$$u(y,t) = (1+K_1)e^{-M_2 y} - K_1 e^{-P_r y}$$

$$+\in\left[(1+K_3+K_5+K_6)e^{-M_4 y} - k_3 e^{-M_2 y} - K_5 e^{-M_3 y} - K_6 e^{-P_r y}\right] e^{-nt}. \qquad ...(23)$$

$$T(y,t) = e^{-\Pr y} + \in\left[\left(1-\frac{P_r}{n}\right)e^{-M_2 y} + \frac{P_r}{n}e^{-P_r y}\right]e^{-nt} \qquad ...(24)$$

SKIN-FRICTION AND HEAT TRANSFER

The skin friction coefficient (?) at the plate at $y = 0$ is:

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = K_8 + \in K_9 e^{-nt} \qquad ...(25)$$

The rate of heat transfer in terms of Nusselt number (N_u) at the vertical at $y = 0$ is:

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = \Pr + \in K_{10} e^{-nt} \qquad ...(26)$$

Where

$$K_8 = \Pr K_1 - (1 + K_1) M_2$$

$$K_9 = K_3 M_2 + K_5 M_3 + K_6 \Pr - (1 + K_3 + K_5 + K_6) M_4$$

and $$K_{10} = \left(1 - \frac{\Pr}{n}\right) M_2 + \frac{\Pr^2}{n}$$

Table 1: Effects of Pr, *M*, *S*, *m*, *K* and *Gr* on skin-friction (t = 1.0, n = 1.0 and $\in$ = 0.005)

Pr	*M*	*K*	*m*	*S*	*Gr*	τ
0.71	1.0	3.0	0.2	1.0	4.0	1.562661
3.0	1.0	3.0	0.2	1.0	4.0	-4.200776
0.71	1.2	3.0	0.2	1.0	4.0	0.575800
0.71	1.0	6.0	0.2	1.0	4.0	2.318750
0.71	1.0	3.0	0.4	1.0	4.0	1.970041
0.71	1.0	3.0	0.2	1.0	5.0	2.270638
0.71	1.0	3.0	0.2	1.1	4.0	1.841697

Table 2: Effects of Pr, *n* and *t* on rate of heat transfer

Pr	*n*	*t*	N_u
0.71	1.0	1.0	0.7106417
3.0	1.0	1.0	3.0010429
0.71	2.0	1.0	0.7103473
0.71	1.0	2.0	0.7102361

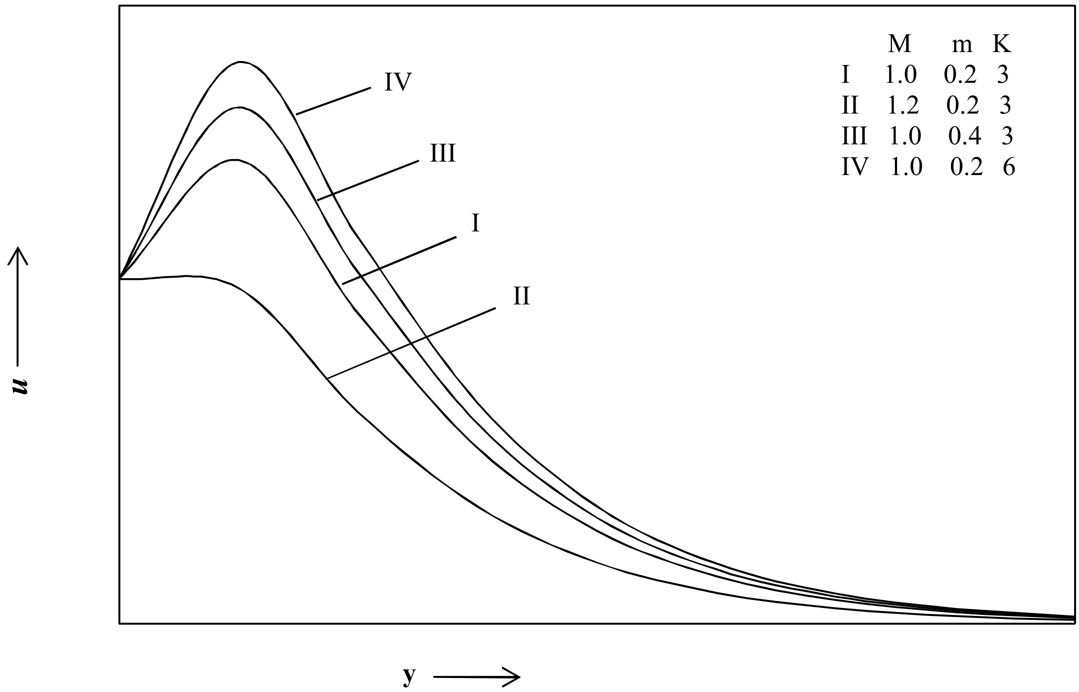

Fig. 1: The velocity profile for the different value of *M*, m and *K*.

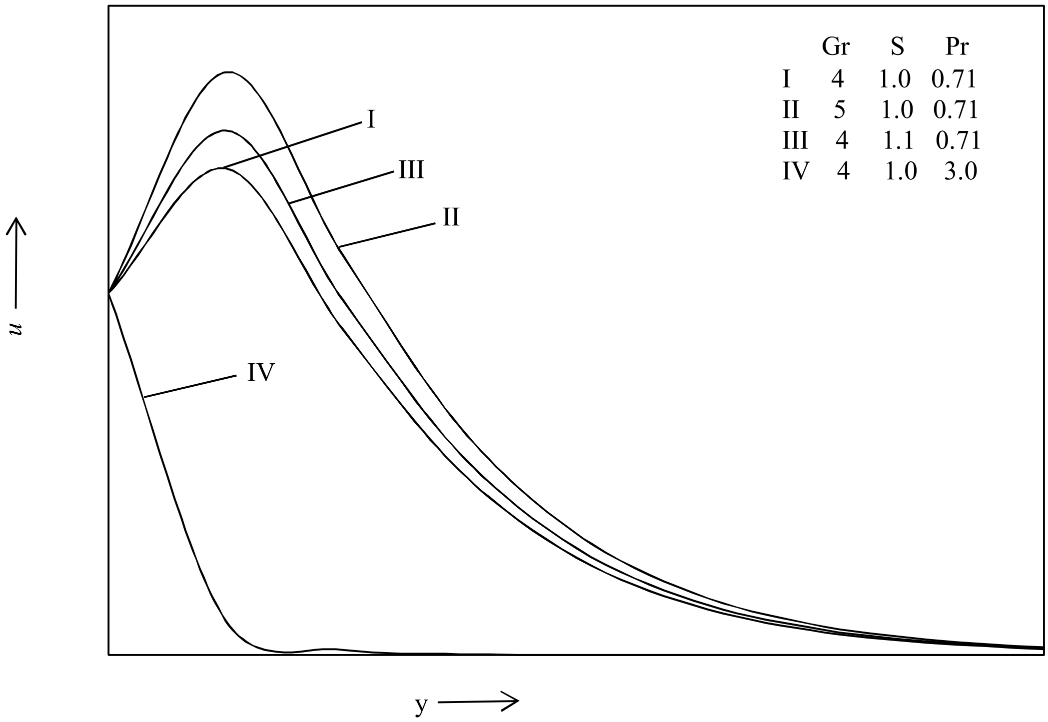

Fig. 2: The velocity profile for different value of *Gr*, *S* and Pr.

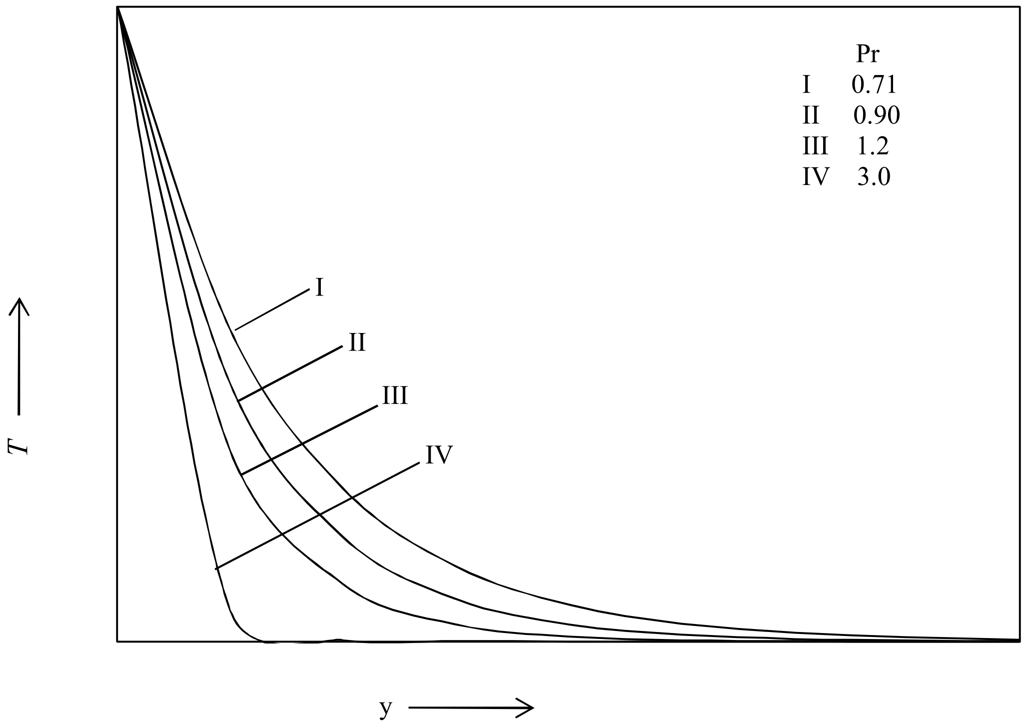

Fig. 3: The temprature profile for different value of Pr.

DISCUSSION AND CONCLUSIONS

It is observed, the physical depth of the problem, the effects of Hall Current parameter (m), magnetic parameter (M), porosity parameter (K), on velocity field are noted time t = 1.0, Prandtl number P_r = 0.71, n = 1.0, stratification factor S =1.0, Grashof number Gr = 3 and $\in$ = 0.005 are depicted in Fig. 1 and the effects of Prandtl number (Pr), Grashof number (Gr) and stratification factor (S) on velocity field at magnetic parameter M=1.0, porosity parameter K=3.0, Hall Current parameter m = 0.2, time t = 1.0, n = 1.0 and $\in = 0.002$ are depicted in Fig. 2. The effect of Prandtl number (Pr) on temperature field (T) is shown in Fig. 3. The effects of the said parameter on skin-friction (τ) are represented in Table 1 and the effects of Prandtl number (Pr), frequency parameter (n) and time parameter (t) on rate of heat transfer (N_u) are represented in Table 2. The conclusions of the study are as follows-

1. An increase in K or m, increases the velocity field while an increase in M, decreases the velocity field.
2. An increase in Gr or S, increases the velocity field while an increase in Pr, decreases the velocity field.
3. An increase in the value of Prandtl number decreases the temperature field.
4. An increase in Pr or M decreases the skin-friction at the plate while on increase S, K, m or Gr increase the skin-friction at the plate.
5. An increase in Pr increases the rate of heat transfer while an increase in n or t decreases the rate of heat transfer.

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