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RESEARCH ARTICLE

A Measurement of Income Inequality Based on Deviation from Maximum Income

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ABSTRACT

Most of the scholars tried to find income inequality by taking mean income. In the present paper an attempt has been made to develop a measure of income inequality by taking maximum income into consideration. Also, to show the computational process a hypothetical data has been used.

Key words: *Income Inequality, Extreme Value, Percentile*

INTRODUCTION

The unequal distribution of income and wealth among members of a region is a universally observed phenomenon. The extent of such inequality can be judged only if we examine the relevant statistical evidence. Such statistical information is usually in short supply and even the available data is often of lower quality than those in many other fields. However, without such information it is impossible to assess the problem and magnitude of inequality. The early estimates are based on social tables. Holmes (1977) showed King's limitation as a social analyst and criticized his social table. The problem of measuring income inequality can be traced back to the end of the last century. Pareto (1895) discussed the topic in a study on personal income distribution, Pareto based his work mainly on fiscal data and interpreted the parameter α of the model he proposed as in income inequality measure. Lorenz (1905) introduced a graphical tool which has since then been called the Lorenz curve and has played an important role in subsequent studies on inequality. Gini (1909) analysed the relationship between social classes and wealth distribution, introduced a parameter 'δ' and argued the 'δ' unlike Pareto's 'α', was a direct measure of concentration.

Gini (1914) arrives at a summary inequality index, called as concentration ratio (R). Atkinson (1970) proposed that R and other conventional summary indices should no longer be used because they do not rank income distribution according to strictly concave social utility functions. Atkinson's point of view stimulated the interest of researchers and was at the origin of many studies on the measurement of income inequality. Dagum (1990) showed how every income inequality measure has a social welfare base and vice-versa.

MEASURES BASED ON MAXIMUM INCOME

Most of the researchers tried to find income inequality by taking mean income. In the resent paper an attempt has been made to develop income inequality by taking maximum income in place of mean income.

Suppose $x_1, x_2, \ldots\ldots. x_n$ be the incomes of 'n' persons such that $x_1 \le x_2 \le \ldots\ldots. \le x_n$. Let R_n be the maximum value, then the deviations taken from this maximum value are $R_n - x_1, R_n - x_2, \ldots.. R_n - x_n$.

Therefore, sum of deviations is:

$$\sum_{i=1}^{n} (R_n - x_i)$$

$I_1 = (nR_n - n\bar{x})$

This may be considered as a measure of inequality. Its value lies between 0 and $nR_n\left(1-\frac{1}{n}\right)$ and hence depends on n and maximum value R_n.

Therefore, after eliminating the effect of n and R_n we get:

$$I_2 = \left(1 - \frac{\bar{x}}{R_n}\right)$$

Clearly, the value of I_2 lies between 0 and $(1 - 1/n)$ which usually happens in case of a good measure of inequality.

If very few incomes are too large in comparison to most of the income, then the proposed measure I_2 will largely effected by R_n.

Therefore to remove such effect, one can take $Z_{(1-\alpha)}$ in place of R_n, where $Z_{(1-\alpha)}$ is the $(1-\alpha)^{th}$ percentile of the income distribution. Here α is usually very small depending on particular cases.

Therefore, the improved index is:

$$I_3 = \left(1 - \frac{\bar{x}}{Z_{(1-\alpha)}}\right)$$

I_3 is a simple index of income inequality. Now, to have a more sensitive index, one must assign large weights to larger income deviations about R_n.

Let W_i be the weight corresponding to $(R_n - x_i)$. It may be assume that sum of weights is unity. The following table supplies the detailed structure:

X	Deviations	Weights	Products
x_1	$R_n - x_1$	w_1	$(R_n - x_1)\,w_1$
x_2	$R_n - x_2$	w_2	$(R_n - x_2)\,w_2$
.	.	.	.
.	.	.	.
.	.	.	.
x_n	$R_n - x_n$	w_n	$(R_n - x_n)\,w_n$
Sum	Unweight sum $\sum_{i=1}^{n} (R_n - x_i)$	1	Weighted sum $\sum_{i=1}^{n} (R_n - x_i)\,w_i$

Now, we propose the following general model of income inequality.

$$I_4 = \phi \sum_{i=1}^{n} (R_n - x_i)\,w_i \quad \text{....(1)}$$

where ϕ is a normalizing parameter to be determined by normalization axiom.

Let $w_i \propto (R_n - x_i)$

$$w_i = k\left(R_n - x_i\right) \quad \text{....(2)}$$

where k is a constant.
And,

$$\sum_{i=1}^{n} w_i = k\sum_{i=1}^{n}\left(R_n - x_i\right)$$

$$\sum_{i=1}^{n} w_i = k\left(nR_n - \sum_{i=1}^{n} x_i\right)$$

Therefore, $k = \dfrac{1}{n\left(R_n - \overline{x}\right)}$ ....(3)

Put the value of k in equation (2), therefore

$$w_i = \frac{\left(R_n - x_i\right)}{n\left(R_n - \overline{x}\right)} \quad \text{....(4)}$$

Put the value of w_i in equation (1), then

$$I_4 = \phi\sum_{i=1}^{n}\left(R_n - x_i\right)\frac{\left(R_n - x_i\right)}{n\left(R_n - \overline{x}\right)}$$

$$I_4 = \phi\sum_{i=1}^{n}\frac{\left(R_n - x_i\right)^2}{n\left(R_n - \overline{x}\right)} \quad \text{....(5)}$$

In case of extreme inequality i.e. $x_i = x_2 = ... = x_{n-1} = 0$ and $x_n = R_n$, we assume the value of inequality as $\left(1 - \frac{1}{n}\right)$.

Now, $\overline{x} = \dfrac{x_n}{n} = \dfrac{R_n}{n}$

From equation (5)

$$I_4 = \phi\left[\frac{R_n^2 + R_n^2 + ... + R_n^2\left(n-1\right)time + \left(R_n - R_n\right)^2}{n\left(R_n - \dfrac{R_n}{n}\right)}\right]$$

Then $\left(1 - \dfrac{1}{n}\right) = \phi\,\dfrac{\left(n-1\right)R_n^2}{nR_n\left(1 - \dfrac{1}{n}\right)}$

and $\phi = \dfrac{1}{R_n}\left(1 - \dfrac{1}{n}\right)$

Hence from equation (5)

$$I_4 = \frac{1}{R_n}\left(1 - \frac{1}{n}\right)\sum_{i=1}^{n}\frac{\left(R_n - x_i\right)^2}{n\left(R_n - \overline{x}\right)}$$

Now the single measure I_4 is:

$$I_4 = \frac{(n-1)}{n^2 R_n} \sum_{i=1}^{n} \frac{(R_n - x_i)^2}{(R_n - \bar{x})}$$

In case, R_n is replaced by $z_{(1-\alpha)}$ as discussed previously, thenthe index becomes:

$$I_5 = \frac{(n-1)}{n^2 z_{(1-\alpha)}} \sum_{i=1}^{n} \frac{\left(z_{(1-\alpha)} - x_i\right)^2}{\left(z_{(1-\alpha)} - \bar{x}\right)}$$

In the above discussion we used deviation from maximum value to find income inequality.

COMPUTATION FOR A HYPOTHETICAL DATA

Now, we consider a hypothetical data to compute some proposed measures. Suppose there is a firm of 50 persons and the monthly salary of these employees are as follows:

2300	2350	2200	2400	3000	3200
3500	2600	2800	2250	2500	3000
3500	5000	5200	2800	2700	3000
3200	2700	4500	5000	6000	5500
5800	10000	12000	13000	14000	13000
12000	18000	16000	15000	19000	17000
14000	13000	16000	16000	18000	5250
11000	13000	16000	30000	32000	35000
50000	3350				

Here the maximum value is $R_n = 50000$

And the mean income of the above data is $\bar{x} = 10376$

Then our proposed index is:

$$I_1 = nR_n - n\bar{x} \quad I_1 = 50 \times 50000 - 50 \times 10376$$

$$I_1 = 2500000 - 518800 \quad I_1 = 1981200$$

and,

$$I_2 = \left(1 - \frac{\bar{x}}{R_n}\right)$$

therefore,

$$I_2 = 1 - \frac{10376}{50000} \; I_2 = 0.79248$$

We can assign weight to all income to increase the effect of this measure by the following formula:

$$I_4 = \frac{(n-1)}{n^2 R_n} \sum_{i=1}^{n} \frac{(R_n - x_i)^2}{(R_n - \bar{x})}$$

Therefore the index is:

$$I_4 = \frac{(50-1)}{50^2 \times 50000} 2101556.38 \quad = 0.823810101$$

Hence we can see from above result that weight index is more effected than simple aggregation index.

Here we see that 5 persons have very high income and if we take deviation from maximum value, then it is not justified. So we will use here 90th percentile.

Then, α = 0.1 i.e. 90 percentile

and $z_{(1-\alpha)}$=19000

therefore I_3 = 0.453894737

Similarly, we can find weighted index.

Hence,

$$I_5 = 0.479045$$

CONCLUSION

In the present paper measure of income inequality I_1 developed in section 2 is based on maximum income instead of based on mean income. Further to make it more reliable and valid under different conditions the proposed measure has been modified with giving weightage to deviation of income from maximum income.

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